Question	Scheme	Marks	AOs
1(a)	$h = 0.5$	B1	1.1b
	$A \approx \frac{1}{2}\times\frac{1}{2}\{0.4805+1.9218+2(0.8396+1.2069+1.5694)\}$	M1	1.1b
	$= 2.41$	A1	1.1b
		(3)	
(b)	$\int(\ln x)^2\ \mathrm{d}x = x(\ln x)^2 - \int x\times\frac{2\ln x}{x}\ \mathrm{d}x$	M1 A1	3.1a 1.1b
	$= x(\ln x)^2 - 2\int \ln x\ \mathrm{d}x = x(\ln x)^2 - 2\left(x\ln x - \int \mathrm{d}x\right)$ $= x(\ln x)^2 - 2\int \ln x\ \mathrm{d}x = x(\ln x)^2 - 2x\ln x + 2x$	dM1	2.1
	$\int_2^4 (\ln x)^2\ \mathrm{d}x = \left[x(\ln x)^2 - 2x\ln x + 2x\right]_2^4$ $= 4(\ln 4)^2 - 2\times4\ln4 + 2\times4 - \left(2(\ln 2)^2 - 2\times2\ln2 + 2\times2\right)$ $= 4(2\ln2)^2 - 16\ln2 + 8 - 2(\ln2)^2 + 4\ln2 - 4$	ddM1	2.1
	$= 14(\ln 2)^2 - 12\ln 2 + 4$	A1	1.1b
		(5)	
		(8 marks)	

Notes

(a)

B1: Correct strip width. May be implied by $\frac{1}{2}\times\frac{1}{2}\{....\}$ or $\frac{1}{4}\times\{....\}$

M1: Correct application of the trapezium rule.

Look for $\frac{1}{2}\times"h"\{0.4805+1.9218+2(0.8396+1.2069+1.5694)\}$ condoning slips in the digits.

The bracketing must be correct but it is implied by awrt 2.41

A1: 2.41 only. This is not awrt

(b)

M1: Attempts parts the correct way round to achieve $\alpha x(\ln x)^2 - \beta\int \ln x\ \mathrm{d}x$ o.e.

May be unsimplified (see scheme). Watch for candidates who know or learn $\int \ln x\ \mathrm{d}x = x\ln x - x$

who may write $\int(\ln x)^2\ \mathrm{d}x = \int(\ln x)(\ln x)\ \mathrm{d}x = \ln x(x\ln x - x) - \int\frac{x\ln x - x}{x}\ \mathrm{d}x$

A1: Correct expression which may be unsimplified

dM1: Attempts parts again to (only condone coefficient errors) to achieve $\alpha x(\ln x)^2 - \beta x\ln x \pm \gamma x$ o.e.

ddM1: Applies the limits 4 and 2 to an expression of the form $\pm\alpha x(\ln x)^2 \pm \beta x\ln x \pm \gamma x$, subtracts and applies $\ln 4 = 2\ln 2$ at least once. Both M's must have been awarded

A1: Correct answer

....

It is possible to do $\int(\ln x)^2\ \mathrm{d}x$ via a substitution $u = \ln x$ but it is very similar.

M1 A1, dM1: $\int u^2\,\mathrm{e}^u\ \mathrm{d}u = u^2\,\mathrm{e}^u - \int 2u\,\mathrm{e}^u\ \mathrm{d}u, = u^2\,\mathrm{e}^u - 2u\mathrm{e}^u \pm 2\mathrm{e}^u$

ddM1: Applies appropriate limits and uses $\ln 4 = 2\ln 2$ at least once to an expression of the form $u^2\,\mathrm{e}^u - \beta u\mathrm{e}^u \pm \gamma\mathrm{e}^u$ Both M's must have been awarded

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Question	Scheme	Marks	AOs
2	$\int x^3 \ln x \, dx = \frac{x^4}{4}\ln x - \int \frac{x^4}{4} \times \frac{1}{x} \, dx$	M1	1.1b
	$= \frac{x^4}{4}\ln x - \frac{x^4}{16}(+c)$	M1 A1	1.1b 1.1b
	$\int_1^{e^2} x^3 \ln x \, dx = \left[\frac{x^4}{4}\ln x - \frac{x^4}{16}\right]_1^{e^2} = \left(\frac{e^8}{4}\ln e^2 - \frac{e^8}{16}\right) - \left(-\frac{1^4}{16}\right)$	M1	2.1
	$= \frac{7}{16}e^8 + \frac{1}{16}$	A1	1.1b
		(5)	
			(5 marks)
Notes:			

M1: Integrates by parts the right way round.

Look for $kx^4 \ln x - \int kx^4 \times \frac{1}{x} \, dx$ o.e. with $k > 0$. Condone a missing dx

M1: Uses a correct method to integrate an expression of the form $\int kx^4 \times \frac{1}{x} \, dx \to c\, x^4$

A1: $\int x^3 \ln x \, dx = \frac{x^4}{4}\ln x - \frac{x^4}{16}$ $(+\, c)$ which may be left unsimplified

M1: Attempts to substitute 1 and e^2 into an expression of the form $\pm px^4 \ln x \pm qx^4$, subtracts and uses $\ln e^2 = 2$ (which may be implied).

A1: $\frac{7}{16}e^8 + \frac{1}{16}$ o.e. Allow $0.4375e^8 + 0.0625$ or uncancelled fractions. NOT ISW: $7e^8 + 1$ is A0

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You may see attempts where substitution has been attempted.

E.g. $u = \ln x \Rightarrow x = e^u$ and $\frac{dx}{du} = e^u$

M1: Attempts to integrate the correct way around condoning slips on the coefficients

$$\int x^3 \ln x \, dx = \int e^{4u} u \, du = \frac{e^{4u}}{4}u - \int \frac{e^{4u}}{4} \, du$$

M1 A1: $\int x^3 \ln x \, dx = \frac{e^{4u}}{4}u - \frac{e^{4u}}{16}(+c)$

M1 A1: Substitutes 0 and 2 into an expression of the form $\pm pue^{4u} \pm qe^{4u}$ and subtracts

....

It is possible to use integration by parts "the other way around"

To do this, candidates need to know or use $\int \ln x \, dx = x\ln x - x$

FYI $I = \int x^3 \ln x \, dx = x^3(x\ln x - x) - \int (x\ln x - x) \times 3x^2 \, dx = x^3(x\ln x - x) - 3I + \frac{3}{4}x^4$

Hence $4I = x^4 \ln x - \frac{1}{4}x^4 \Rightarrow I = \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4$

Score M1 for a full attempt at line 1 (condoning bracketing and coefficient slips) followed by M 1 for line 2 where terms in I o.e. to form the answer.